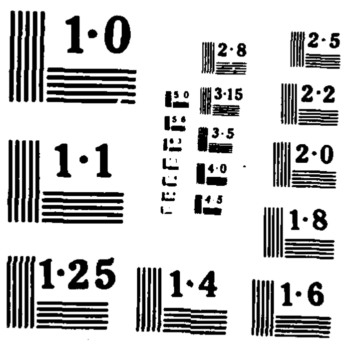
AD-A169 926 HEREDITY OF STATIONARY AND REVERSIBLE STOCHASTIC PROCESSES(U) GEORGIA INST OF TECH ATLANTA R F SERFOZO 25 MAR 86 AFOSR-TR-86-0381 AFOSR-84-0367 1/1

UNCLASSIFIED F/G 12/1 NL


1·0
2·8
2·5
3·15
2·2
3·5
1·1
2·0
4·0
4·5
1·8
1·25
1·4
1·6

AD-A169 926

(2)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY NA	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for Public Release; Distribution Unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE NA	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 86-0381
6a. NAME OF PERFORMING ORGANIZATION Georgia Institute of Technology 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION AFOSR/NM
6c. ADDRESS (City, State and ZIP Code) Atlanta, Georgia 30332	7b. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR 84-0367
8c. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO.: 6.1102F PROJECT NO.: 2304 TASK NO.: A5 WORK UNIT NO.:
11. TITLE (Include Security Classification) Heredity of Stationary and Reversible Stochastic Processes	
12. PERSONAL AUTHOR(S)	

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Preprint	FROM ______ TO ______	March 25, 1986	5

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

When a stochastic process (a random measure, set, field, etc. on a group) is stationary, ergodic, or reversible, then certain functions of this process inherit these properties. We present sufficient conditions for this inheritance.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Richard F. Serfozo	22b. TELEPHONE NUMBER (Include Area Code) (404)894-2305 22c. OFFICE SYMBOL

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE. UNCLASSIFIED

86

AFOSR-TR- 86-0381

Heredity of Stationary and Reversible Stochastic Processes

by

Richard F. Serfozo
Georgia Institute of Technology

Abstract. When a stochastic process (a random measure, set, field, etc. on a group) is stationary, ergodic, or reversible, then certain functions of this process inherit these properties. We present sufficient conditions for this inheritance.

Introduction. Associated with a queueing network process (see Chapters 2,3 of Kelly (1979)), there are a number of stochastic processes that describe various aspects of the queues. Examples are (i) the point process of times at which customers move from a certain set of queues to another set, and (ii) the process depicting the service station with the largest queue and the length of that queue over time. When the queueing network process is stationary or reversible, do these associated processes inherit these properties? I shall present general criteria for this inheritance. The basic issue is: If a stochastic process is stationary or reversible, then what types of functions of it are also stationary or reversible?

Preliminaries. Let $X = \{X(t): t \in R\}$ be a stochastic process with state space S. Here R denotes the real line or, more generally, a group with a σ-field $\mathcal{R}$ on it that renders the group operation (addition) measurable. Assume that X has sample paths in a subspace $\mathcal{X}$ of the space $\underset{\sim}{F}(R,S)$ of all measurable functions from R to S. The process X is

---

This research was supported in part by grant AFOSR-84-0367.

stationary if, for any $s_1,\ldots,s_n$ in R,

$$X(s_1 + t),\ldots,X(s_n + t) \stackrel{d}{=} X(s_1),\ldots,X(s_n),\ t\varepsilon R,$$

(here $\stackrel{d}{=}$ means equality in distribution). A compact way of expressing this is $X \circ T_t \stackrel{d}{=} X$, $t \varepsilon R$, where $\circ$ denotes the convolution operator and T_t is the time-shift transformation on R ($T_t s = s + t$). A stationary process X is ergodic if each of its invariant events has probability zero or one (an event A is invariant for X if there is a $B \varepsilon \mathcal{X}$ such that $\{X \circ T_t \varepsilon A\} = B$, $t \varepsilon R$). The process X is reversible if it is stationary and $X \circ T^- \stackrel{d}{=} X$, where T^- is the time-reversal transformation on R ($T^- t = -t$). Note that X is reversible if and only if $X \circ T_t \circ T^- \stackrel{d}{=} X$, $t\varepsilon R$ (this is sometimes used as the definition).

The preceding terminology also applies to point processes and other random elements. Let $N = \{N(B): B \varepsilon \mathcal{R}\}$ be a point process on R, where N(B) represents the number of points (possibly infinite) in the set B. Stationarity, ergodicity, and reversibility of N are defined as above, where T_t and T^- are the time-shift and time-reversal transformations on $\mathcal{R}$ (instead of R): $T_t B = B + t$, $T^- B = -B$. For instance, N is reversible if it is stationary ($N \circ T_t \stackrel{d}{=} N$, $t \varepsilon R$) and $N \circ T^- \stackrel{d}{=} N$. There are similar definitions of stationarity, ergodicity, and reversibility for random measures, random sets, etc., and vectors of these elements.

It is well known that if the process X is stationary and $g: S \rightarrow S'$ and $h: \mathcal{X} \rightarrow \underset{\sim}{F}(R,S')$ are measurable functions, then the processes $Y(t) = g(X(t))$ and $Z(t) = h(X \circ T_t)$ are also stationary. (The g is a special case of h.) Moreover, Y and Z are ergodic when X is. See for instance

Chapter 6 of Breiman (1968). In addition, an easy check shows that Y and Z are reversible when X is. I shall now show how these results extend to a larger class of functions and to point processes and other random elements.

Results. A key observation is that stationarity and reversibility are special cases of the following notion of invariance of a process under a family of "time" transformations. Let Y be a random element and let Φ be a family of measurable transformations from the domain of Y onto itself. The domain of Y is R or $\mathcal{R}$ or cartesion products of them. The random element Y is invariant under the set of transformations Φ if $Y \circ \phi \overset{d}{=} Y$ for each $\phi \in \Phi$. For example, the vector $Y = (X,N)$ is reversible if it is invariant under the transformations $\{T^-, T_t\colon\ t \in R\}$, where T^- and T_t are the time-reversal and time-shift transformations on $R \times \mathcal{R}$.

For the following results, let Y and Z denote random elements (possibly vectors of processes, measures, sets, etc.) with sample paths or realizations in the respective spaces $\mathcal{Y}$ and $\mathcal{Z}$. Assume that $Z = f(Y)$, where $f\colon \mathcal{Y} \to \mathcal{Z}$ is a measurable function.

Lemma. If Y is invariant under the set of transformations Φ, and f satisfies $f(y) \circ \phi = f(y \circ \phi)$, for each $\phi \in \Phi$, $y \in \mathcal{Y}$, then Z is invariant under the set of transformations Φ.

Proof. This follows since, for each $\phi \in \Phi$,

$$Z \circ \phi = f(Y) \circ \phi = f(Y \circ \phi) \overset{d}{=} f(Y) = Z.$$

Proposition. If Y is stationary and f satisfies

(1) $$f(y) \circ T_t = f(y \circ T_t), \text{ for each } t \in R,\ y \in \mathcal{Y},$$

then Z is stationary. If Y is reversible and f satisfies (1) and $f(y) \circ T^- = f(y \circ T^-)$, $y \in \mathcal{Y}$, then Z is reversible. If Y is stationary and ergodic and f satisfies (1), then Z is stationary and ergodic.

Proof. The first two assertions are special cases of the preceding lemma. The third assertion follows since $\{Z \circ T_t \in A\} = \{Y \circ T_t \in f^{-1}(A)\}$ implies that the invariant events of Z are the same as those for Y.

Example. Suppose $Y = \{(Y_1(t),\ldots,Y_m(t)): t \in R\}$ is a queueing network process, where $Y_j(t)$ denotes the number of customers at queue j at time t. Assume that Y cannot have an infinite number of transitions in a finite time interval. Specifics on the operation of the queues are not needed for this discussion. Let $Y_J(t) = \sum_{j \in J} Y_j(t)$, the number of customers in the set of queues $J \subset \{1,\ldots,m\}$. Let N denote the point process of times at which customers move from J to another set of queues K, and let X(t) denote the largest number of customers in J since the last time the number was zero. Clearly $(N,X) = f(Y) = (f_1(Y), f_2(Y))$, where

$$f_1(y) = \sum_{t \in B} I(Y_J(t) = Y_J(t-) - 1,\ Y_K(t) = Y_K(t-) + 1),$$

I is the indicator function, and

$$f_2(y)(t) = \max\{y_J(s): \tau_t < s < t\},\ t \in R,$$

and $\tau_t = \sup\{s < t: y_J(s) = 0\}$, assuming that τ_t exists for each t. Then the preceding proposition establishes that the vector (N,X) is stationary, ergodic or reversible when Y has these respective properties.

Extensions. The lemma extends to state-space transformations as well as time transformations as follows. Define the random element Y to be (h,Φ)-invariant if $h(Y \circ \phi) \stackrel{d}{=} h(Y) \circ \phi$, $\phi \in \Phi$, where $h: \mathcal{Y} \to \underset{\sim}{F}(R,S')$ is measurable. For example, if the process X is $(h,\{T_t : t \in R\})$-invariant, where $h(x)(t) = g(x(t))$ and $g: S \to S$ satisfies $g(g(s)) = s$, then X is dynamically reversible (see p. 31 of Kelly (1979)). The lemma for (h,Φ)-invariance is: If Y is (h,Φ)-invariant and f satisfies $h(f(y) \circ \phi) = h(f(y \circ \phi))$, $\phi \in \Phi$, $y \in \mathcal{Y}$, then $Z = f(Y)$ is also (h,Φ)-invariant. The preceding results readily extend to time and space transformations h,Φ that are random.

References

Breiman, L. (1968). Probability. Addison-Wesley.

Kelly, F.P. (1979). Reversibility and Stochastic Networks. John Wiley.

END

DTIC

8-86

END

DTIC

8-86